MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2015/2016

EEM3066 – RANDOM PROCESSES & QUEUEING THEORY

(TE)

8 OCTOBER 2015

9:00 A.M – 11:00 A.M.

(2 Hours)

INSTRUCTIONS TO STUDENT:

1. This exam paper consists of **9 pages** with **4 questions**.

2. Attempt **ALL questions**. All questions have equal weightage and the breakdown of marks within each question is given.

3. Please write all your answers in the answer booklet provided. Show all relevant steps to obtain maximum marks

4. Some common trigonometric identities, Fourier transform pairs, Fourier transform properties and the Erlang-B table are supplied in the **Appendix**.

Question 1

(a) Each front tyre of a vehicle is supposed to be filled to a pressure of 1.7 bar. Suppose the actual air pressure in each tyre is a random variable, X for the right tyre and Y for the left tyre, with joint pdf

$$f(x,y) = \begin{cases} k(x^2+y^2) & 1.4 \le x \le 2, 1.4 \le y \le 2 \\ 0 & elsewhere \end{cases}$$

i. What is the value of k?

[4 marks]

ii. What is the probability that the air pressure of the right tyre is lower than the one of the left tyre?

[6 marks]

iii. Determine the marginal distribution of air pressure in the right tyre alone.

[3 marks]

iv. What is the probability that both tyres are underfilled?

[4 marks]

(b) A simple model (in degrees Celsius) for the daily temperature process is given by

$$T_n = 16\left[1 - cos\left(\frac{2\pi n}{365}\right)\right] + 4X_n$$

where $X_1, X_2, \ldots$ is an iid random sequence of Gaussian random variables with zero mean and unit variance.

i. What is the mean $E[T_n]$?

[4 marks]

ii. Find the autocovariance function $C_T(n, n+k)$.

[4 marks]

Continued...

Question 2

Consider the linear system shown in Figure Q2.

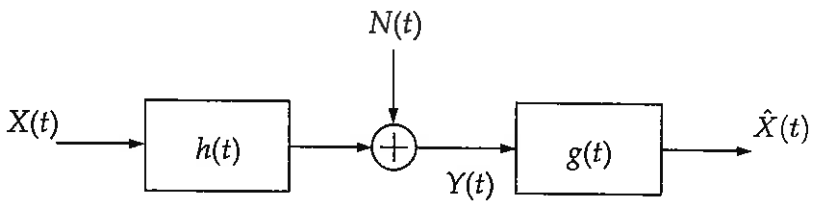

Figure Q2

The system is excited by two *orthogonal* zero-mean, jointly wide-sense stationary random processes $X(t)$, the signal, and $N(t)$, the noise. Let the input to the system $g(t)$ be

$$Y(t) = h(t) * X(t) + N(t)$$

where $*$ denotes convolution. If we pass the received signal $Y(t)$ through the filter $g(t)$, we obtain an estimate $\hat{X}(t)$. Finally, we define the estimation error $\varepsilon(t)$ as

$$\varepsilon(t) = \hat{X}(t) - X(t)$$

i. Let $U(t) = h(t) * X(t)$. Show that $U(t)$ and $N(t)$ are orthogonal.

[4 marks]

ii. Find $S_{YY}(f)$ in terms of $H(f)$, $S_{XX}(f)$ and $S_{NN}(f)$.

[5 marks]

iii. Find $S_{\hat{X}X}(f)$ in terms of $H(f)$, $G(f)$ and $S_{XX}(f)$.

[7 marks]

iv. Find $S_{\varepsilon\varepsilon}(f)$ in terms of $H(f)$, $G(f)$, $S_{XX}(f)$ and $S_{NN}(f)$.

[6 marks]

v. Show that, if $S_{NN}(f) \gg S_{XX}(f)$, in order to minimize $S_{\varepsilon\varepsilon}(f)$, we should have $G(f) \approx 0$.

[3 marks]

Continued...

Question 3

(a) The received signal in a binary communication system that employs antipodal signals is

$$r(t) = s(t) + n(t)$$

where $s(t)$ is shown in Figure Q3 and $n(t)$ is AWGN with power spectral density $N_0/2$ W/Hz.

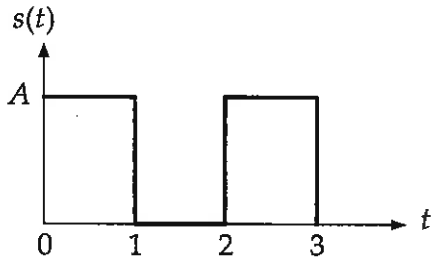

Figure Q3

i. Sketch carefully the impulse response of the *causal* filter matched to $s(t)$.

[4 marks]

ii. Sketch carefully the output of the matched filter when the input is $s(t)$.

[6 marks]

iii. Determine the variance of the noise at the output of the matched filter at $t = 3$.

[7 marks]

iv. Find the maximum SNR at the output of the matched filter.

[3 marks]

(b) Find the transfer function of a Wiener filter for a random signal $X(t)$ corrupted by uncorrelated white noise $N(t)$ with power spectral density $N_0/2$. The autocorrelation function of the signal $X(t)$ is given by

$$R_{XX}(\tau) = \frac{1}{2}e^{-|\tau|}$$

[5 marks]

Continued...

Question 4

Two computers need to be connected to a router. The connection is achieved by using two separate lines as shown in Figure Q4. Suppose that each computer generates λ messages per second and that the average time for message delivery is given as $1/\mu$ for the individual lines.

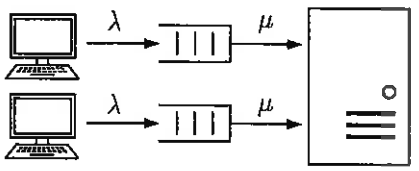

Figure Q4

In this configuration, there are essentially two separate M/M/1 queueing systems with the same performance.

i. Draw the transition rate diagram for one of the transmission lines.

[4 marks]

ii. Show that the probability of having j messages in a transmission line is given by

$$p_j = (1-\rho)\rho^j \qquad \text{for } j = 0,1,2,\ldots$$

where $\rho = \lambda/\mu$

$$\left[Hint: \sum_{i=0}^{\infty} x^i = \frac{1}{1-x} \text{ for } |x| < 1\right]$$

[11 marks]

iii. Show that the average number of messages within each of the transmission lines is given by

$$E[N] = \frac{\rho}{1-\rho}$$

$$\left[Hint: \sum_{i=0}^{\infty} ix^i = \frac{x}{(1-x)^2} \text{ for } |x| < 1\right]$$

[4 marks]

iv. Throughput is defined as $R = 1/E[T]$, where $E[T]$ is the average time required for the message delivery. Using Little's formula, derive an expression for $E[T]$, then find the total throughput of this configuration.

[6 marks]

Continued...

APPENDIX A: Common Trigonometric Identities

$$\cos(x \pm y) = \cos(x)\cos(y) \mp \sin(x)\sin(y)$$

$$\sin(x \pm y) = \sin(x)\cos(y) \pm \cos(x)\sin(y)$$

$$\cos\left(x \pm \frac{\pi}{2}\right) = \mp\sin(x)$$

$$\sin\left(x \pm \frac{\pi}{2}\right) = \pm\cos(x)$$

$$\cos(2x) = \cos^2(x) - \sin^2(x)$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$2\cos(x) = e^{jx} + e^{-jx}$$

$$2j\sin(x) = e^{jx} - e^{-jx}$$

$$2\cos(x)\cos(y) = \cos(x-y) + \cos(x+y)$$

$$2\sin(x)\sin(y) = \cos(x-y) - \cos(x+y)$$

$$2\sin(x)\cos(y) = \sin(x-y) + \sin(x+y)$$

$$2\cos^2(x) = 1 + \cos(2x)$$

$$2\sin^2(x) = 1 - \cos(2x)$$

Continued...

APPENDIX B: Common Fourier Transform Pairs

Time	Frequency
$\delta(t)$	1
1	$\delta(f)$
$u(t)$	$\frac{1}{2}\delta(f)+\frac{1}{j2\pi f}$
$sign(t)$	$\frac{1}{j\pi f}$
$\exp(j2\pi f_0 t)$	$\delta(f-f_0)$
$\exp(-\alpha t)u(t)$	$\frac{1}{\alpha+j2\pi f}$
$\exp(-\alpha\lvert t\rvert)$	$\frac{2\alpha}{\alpha^2+(2\pi f)^2}$
$t\exp(-\alpha t)u(t)$	$\left(\frac{1}{\alpha+j2\pi f}\right)^2$
$\frac{t^{n-1}}{(n-1)!}\exp(-\alpha t)u(t)$	$\left(\frac{1}{\alpha+j2\pi f}\right)^n$
$\exp(-\alpha^2 t^2)$	$\sqrt{\pi/\alpha^2}\exp\left(-\frac{\pi^2 f^2}{\alpha^2}\right)$
$\cos(2\pi f_0 t)$	$\frac{1}{2}[\delta(f-f_0)+\delta(f+f_0)]$
$\sin(2\pi f_0 t)$	$\frac{1}{2j}[\delta(f-f_0)-\delta(f+f_0)]$
$A\,rect\left(\frac{t}{\tau}\right)$	$A\tau\,sinc(f\tau)$
$sinc(2Wt)$	$\frac{1}{2W}rect\left(\frac{f}{2W}\right)$

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APPENDIX C: Fourier Transform Properties

Operation	Time	Frequency
Addition	$x_1(t) + x_2(t)$	$X_1(f) + X_2(f)$
Scalar multiplication	$kx(t)$	$kX(f)$
Duality/Symmetry	$X(t)$	$x(-f)$
Scaling	$x(at)$	$\frac{1}{\lvert a\rvert} X\left(\frac{f}{a}\right)$
Time shift	$x(t - t_0)$	$X(f)\exp(-j2\pi f t_0)$
Frequency shift	$x(t)\exp(j2\pi f_0 t)$	$X(f - f_0)$
Time convolution	$x_1(t) * x_2(t)$	$X_1(f)X_2(f)$
Frequency convolution	$x_1(t)x_2(t)$	$X_1(f) * X_2(f)$
Time differentiation	$\frac{d^n}{dt^n} x(t)$	$(j2\pi f)^n X(f)$
Time integration	$\int_{-\infty}^{t} x(t)dt$	$\frac{X(f)}{j2\pi f} + \frac{1}{2}X(0)\delta(f)$
Conjugation	$x^*(t)$ $x^*(-t)$	$X^*(-f)$ $X^*(f)$

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APPENDIX D: Erlang-B Table

Maximum Offered Load Versus B and N
B is in %

N/B	0.01	0.05	0.1	0.5	1.0	2	5	10	15	20
1	.0001	.0005	.0010	.0050	.0101	.0204	.0526	.1111	.1765	.2500
2	.0142	.0321	.0458	.1054	.1526	.2235	.3813	.5954	.7962	1.000
3	.0868	.1517	.1938	.3490	.4555	.6022	.8994	1.271	1.603	1.930
4	.2347	.3624	.4393	.7012	.8694	1.092	1.525	2.045	2.501	2.945
5	.4520	.6486	.7621	1.132	1.361	1.657	2.219	2.881	3.454	4.010
6	.7282	.9957	1.146	1.622	1.909	2.276	2.960	3.758	4.445	5.109
7	1.054	1.392	1.579	2.158	2.501	2.935	3.738	4.666	5.461	6.230
8	1.422	1.830	2.051	2.730	3.128	3.627	4.543	5.597	6.498	7.369
9	1.826	2.302	2.558	3.333	3.783	4.345	5.370	6.546	7.551	8.522
10	2.260	2.803	3.092	3.961	4.461	5.084	6.216	7.511	8.616	9.685
11	2.722	3.329	3.651	4.610	5.160	5.842	7.076	8.487	9.691	10.86
12	3.207	3.878	4.231	5.279	5.876	6.615	7.950	9.474	10.78	12.04
13	3.713	4.447	4.831	5.964	6.607	7.402	8.835	10.47	11.87	13.22
14	4.239	5.032	5.446	6.663	7.352	8.200	9.730	11.47	12.97	14.41
15	4.781	5.634	6.077	7.376	8.108	9.010	10.63	12.48	14.07	15.61
16	5.339	6.250	6.722	8.100	8.875	9.828	11.54	13.50	15.18	16.81
17	5.911	6.878	7.378	8.834	9.652	10.66	12.46	14.52	16.29	18.01
18	6.496	7.519	8.046	9.578	10.44	11.49	13.39	15.55	17.41	19.22
19	7.093	8.170	8.724	10.33	11.23	12.33	14.32	16.58	18.53	20.42
20	7.701	8.831	9.412	11.09	12.03	13.18	15.25	17.61	19.65	21.64
21	8.319	9.501	10.11	11.86	12.84	14.04	16.19	18.65	20.77	22.85
22	8.946	10.18	10.81	12.64	13.65	14.90	17.13	19.69	21.90	24.06
23	9.583	10.87	11.52	13.42	14.47	15.76	18.08	20.74	23.03	25.28
24	10.23	11.56	12.24	14.20	15.30	16.63	19.03	21.78	24.16	26.50
25	10.88	12.26	12.97	15.00	16.13	17.51	19.99	22.83	25.30	27.72
26	11.54	12.97	13.70	15.80	16.96	18.38	20.94	23.89	26.43	28.94
27	12.21	13.69	14.44	16.60	17.80	19.27	21.90	24.94	27.57	30.16
28	12.88	14.41	15.18	17.41	18.64	20.15	22.87	26.00	28.71	31.39
29	13.56	15.13	15.93	18.22	19.49	21.04	23.83	27.05	29.85	32.61
30	14.25	15.86	16.68	19.03	20.34	21.93	24.80	28.11	31.00	33.84

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